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Slowly varying tachyon and tachyon potential

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Abstract: We show that the scattering amplitude of four open string scalars or tachyons on the world-volume of a Dp-brane in the bosonic string theory can be written in a universal form. The difference between this amplitude and the corresponding amplitude in the superstring theory is in an extra tachyonic pole. We show that in an α' expansion and for slowly varying fields, the amplitude is consistent with the tachyonic DBI action in which the even part of the tachyon potential is $V(T) = e^{-(\sqrt{\pi}T/\alpha)^2}$ with $\alpha = 1$ for bosonic theory and $\alpha = \sqrt{2}$ for superstring theory.

Keywords: Bosonic Strings, D-branes, Tachyon Condensation.

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Contents

1. Introduction

Decay of unstable branes is an interesting process which might shed some new light in understanding properties of string theory in time-dependent backgrounds [1]–[15]. In particular, it was pointed out by Sen that an effective action of the Born-Infeld type proposed in [16]–[18] can capture many properties of these decaying processes [3]. This action for tachyon and for the transverse scalar fields is [16]–[19]:

$$S = -T_p \int d^{p+1}x V(T) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' \partial_a \Phi^i \partial_b \Phi_i + 2\pi\alpha' \partial_a T \partial_b T)}\,, \tag{1.1}$$

where $V(T)$ is the tachyon potential which is one at unstable vacuum $T = 0$ and should be zero at the true stable vacuum. Choosing for the potential [20, 21, 15, 22, 23]

$$V(T) = \frac{1}{\cosh(\sqrt{\pi} T/\alpha)}\,, \tag{1.2}$$

with $\alpha = 1$ for the bosonic theory and $\alpha = \sqrt{2}$ for the superstring theory, one finds from above action the correct stress-tensor in the homogenous time-dependent tachyon condensation which is a tachyon solution that starts at the top of the potential at $x^0 \to -\infty$ and then rolls toward the true vacuum [3, 9, 15]. More over, it has been shown that the tachyon action with the potential (1.2) can be reproduced by nearly on-shell S-matrix elements of slowly varying tachyon vertex operators in the above time-dependent background [22].

It has been shown in [24] that all S-matrix elements involving four scalar and/or tachyon vertex operators in the background $T = 0$ in the superstring theory can be written in a universal form. The leading terms of the α' expansion of this amplitude produces correctly the couplings of four slowly varying scalar or tachyon fields as well as a potential for tachyon which is consistent with the following potential:

$$V(T) = e^{-(\sqrt{\pi} T/\alpha)^2}\,, \tag{1.3}$$

with $\alpha = \sqrt{2}$. In this paper, we extend this calculation to the bosonic theory and show that the even part of the tachyon potential in this case is again consistent with (1.3) with $\alpha = 1$.

As a warm up in this introduction, we discuss the S-matrix element of one closed string tachyon vertex operator and two open string scalar or tachyon vertex operators. These amplitudes are calculated in [25]. They are:[1]

$$
\begin{aligned}
A(\tau,\Phi,\Phi) &\sim \eta_{ij}\zeta_1^i\zeta_2^j\frac{\Gamma(-1-2s)}{\Gamma(-s)\Gamma(-s)}\,,\\
A(\tau,T,T) &= \frac{\Gamma(-1-2s)}{\Gamma(-s)\Gamma(-s)}\,,
\end{aligned}
$$

where τ is the closed string tachyon. In above equations $s=-\alpha'(k_1+k_2)^2/2$, ζ_1^i,ζ_2^i are the scalar polarization vectors in the $(26-(p+1))$-dimensional transverse space, and k_1^a,k_2^a are the momentum vectors in the $(p+1)$-dimensional world-volume space.[2] The momenta in this amplitude satisfy the on-shell condition $k^2=0$ for the scalar and $k^2=1/\alpha'$ for the tachyon. Now as can be seen, the two amplitudes has the same dependency on the Mandelstam variable s. To write both in a universal form, we introduce the polarization vectors $\zeta_1^\alpha,\zeta_2^\beta$ in a $(27-(p+1))$-dimensional space with ζ^{26} polarization of the tachyon. In terms of these new polarization vectors, the universal amplitude is,

$$
A(\zeta_1,\zeta_2)\sim\zeta_1\!\cdot\!\zeta_2\left(\frac{\Gamma(-1-2s)}{\Gamma(-s)\Gamma(-s)}\right)\,. \tag{1.4}
$$

The metric used to multiply the polarizations is $\eta^{\alpha\beta}$. Note that if we do not use the on-shell condition for the momenta, the amplitude in this form is invariant under $\mathrm{SO}(p+1)\times\mathrm{SO}(27-(p+1))$. Comparing this amplitude with the corresponding amplitude in the superstring theory [24], one finds the following relation:

$$
A^{\text{bosonic string}}=\frac{-1}{1+2s}A^{\text{suprstring}}\,. \tag{1.5}
$$

We are interested in the $\alpha'\to 0$ limit of S-matrix elements. To do this expansion, one should first off-shell extend the amplitude, i.e., not using on-shell conditions for the momenta, then expand the gamma functions. At the end one restricts the momenta to the on-shell conditions. The on-shell amplitudes have in general two kinds of terms upon expanding the amplitude at this limit. One includes terms like $k_i\!\cdot\!k_j$ that do not depend on the on-shell condition, and the other kind includes term like k_i^2 that highly depend on the on-shell condition. Only these latter terms are different for scalars and for tachyons. So one may separate the amplitude after α' expansion into two parts, i.e., $A=A'+A^{\text{on}}$. The first part is the same for scalar and tachyon, whereas, the second part depends on open string states, e.g.,$A^{\text{on}}=0$ when all states are scalars. Our speculation [16, 26, 24] is that while the leading α' order terms of A for only scalars are correspond to DBI action, the same A for scalars and/or tachyons are correspond to tachyonic DBI action (1.1) with potential (1.3).

[1]Using the fact that the closed strings are not functional of the open string tachyon but are functional of the scalar fields [30], we assumed that the closed string field does not depend on the scalar. This makes similar the S-matrix elements involving the scalars and the tachyon as much as possible.

[2]Our index conventions are that early Latin indices take values in the world-volume, e.g.,$a,b=0,1,\ldots,p$, middle Latin indices take values in the transverse space, e.g.,$i,j=p+1,\ldots,24,25$, early Greek indices take values in the (27-(p+1))-dimensional space, e.g.,$\alpha,\beta=p+1,\ldots,24,25,26$, and middle Greek indices take value in the whole 26-dimensional space of bosonic theory, e.g.,$\mu,\nu=0,1,\ldots,24,25$

To α' expand the amplitude (1.4), one should first extend it to off-shell physics. One may write the off-shell amplitude as the following:

$$A^{\rm off} \sim \zeta_1\cdot\zeta_2\left(\left(\frac{\Gamma(-1-2s)}{\Gamma(-s)\Gamma(-s)}-G\right)+G\right), \qquad (1.6)$$

where the momenta in this action do not satisfy the on-shell condition. The function $G(k_1,k_2)$ has to cancel non-desirable terms producing by expansion of the gamma functions in the $\alpha'\to 0$ limit, i.e., the terms in the first parenthesis being independent of k_1^2, or k_2^2. Since we are interested in slowly varying fields we continue our discussion for the leading term of expansion. For this term the function G should be i.e.,

$$G=-\alpha'\frac{(k_1^2+k_2^2)}{4}+\cdots .$$

Now α' expand the gamma functions in the amplitude, and then go back to the on-shell physics, i.e.,

$$A \sim \zeta_1\cdot\zeta_2\left(\left(-\alpha'\frac{k_1\cdot k_2}{2}+\cdots\right)+G^{\rm on}\right), \qquad (1.7)$$

where dots represent non-leading terms. The function $G^{\rm on}$ is G in which the momenta are on-shell, i.e., $G^{\rm on}=0$ when the two states are scalars and $G^{\rm on}=-1/2$ when the two states ate tachyons. Now compare it with expansion of the tachyonic DBI action (1.1).[3] The action (1.1) has the following expansion:

$$S=-T_p\int d^{p+1}x\,\tau\left(1-\pi T^2+\pi\alpha'(\partial T)^2+\pi\alpha'(\partial\Phi)^2\cdots\right),$$

where we have used the expansion $V(T)=1-\pi T^2+O(T^3)$ at the top of the tachyon potential in this action. When the polarization vectors in (1.7) are in the transverse directions, i.e., in the $(26-(p+1))$-dimensional space, the first term of (1.7) reproduces the kinetic term of the scalar fields. When the polarization vectors are in the 27-th direction, the first term is reproduced by the kinetic term of tachyon, and $G^{\rm on}$ term is reproduced by the tachyon potential. In the present paper we would like to extend these calculation to the case of S-matrix elements of four tachyon and/or scalar vertex operators.

In next section , we describe the calculation of the scattering amplitudes in the bosonic string theory, and write the results in a universal amplitude which is invariant under $\mathrm{SO}(p+1)\times\mathrm{SO}(27-(p+1))$ when the momenta are off-shell. Then in the section 3, using the idea in [24], we off-shell extend the amplitude, expand it in the limit $\alpha'\to 0$, and then back to the on-shell amplitude. Then compare the leading terms of the expansion with the action (1.1) and find the coefficient of T^4 in the expansion of tachyon potential in this action.

[3]We have used the fact that the closed string tachyon normalizes the D-brane tension as $T_p\to T_p(1+\tau+\cdots)$[25], and we have kept only linear term for the closed string tachyon.

2. Scattering amplitude

Scattering amplitude of four scalar or tachyon vertex operators in the bosonic string theory is given by the following correlation function:

$$A \sim \int dx_1 dx_2 dx_3 dx_4 \langle : V(2{\cdot}k_1, x_1) : V(2{\cdot}k_2, x_2) : V(2{\cdot}k_3, x_3) : V(2{\cdot}k_4, x_4) : \rangle \,, \tag{2.1}$$

where

$$V^{\text{tachyon}} = \zeta^{26} e^{2k\cdot X} \,,$$

$$V^{\text{scalar}} = \zeta^{i} \partial X_i e^{2k\cdot X} \,,$$

with the world-sheet propagator

$$\langle X^{\mu}(x) X^{\nu}(y) \rangle = -\alpha' \eta^{\mu\nu} \ln(x-y)/2 \,, \tag{2.2}$$

where we have used the doubling trick to work with only holomorphic functions on the boundary of world-sheet [27]. Using the Wick theorem, one can easily calculate the correlators in (2.1). The different between the tachyon and scalar vertex operators is in the extra factor of ∂X^i. However, because the momenta in the vertex operators are in the world-volume directions and index i takes value in the transverse space, the factors ∂X^i correlate only among themselves resulting contraction of the scalar polarizations. Performing the correlation, one finds that the integrand has $\mathrm{SL}(2,\mathbb{R})$ symmetry. One should fix this symmetry by fixing position of three vertices in the real line. Different fixing of these positions give different ordering of the four vertices in the boundary of the world-sheet. One should add all non-cyclic permutation of the vertices to get the correct scattering amplitude. So one should add the amplitudes resulting from the fixing $(x_1 = 0, x_2, x_3 = 1, x_4 = \infty)$, $(x_1 = 0, x_2, x_4 = 1, x_3 = \infty)$, $(x_1 = 0, x_3, x_4 = 1, x_2 = \infty)$, $(x_1 = 0, x_3, x_2 = 1, x_4 = \infty)$, $(x_1 = 0, x_4, x_2 = 1, x_3 = \infty)$, $(x_1 = 0, x_4, x_3 = 1, x_2 = \infty)$. After these gauge fixing, one ends up with only one integral which gives the beta function. The resulting amplitude for scalars and tachyons seems different, however, using the conservation of momenta, one can write them all in a universal form:

$$A(\zeta_1,\zeta_2,\zeta_3,\zeta_4) = A_s(\zeta_1,\zeta_2,\zeta_3,\zeta_4) + A_u(\zeta_1,\zeta_2,\zeta_3,\zeta_4) + A_t(\zeta_1,\zeta_2,\zeta_3,\zeta_4) \,,$$

where A_s, A_u, and A_t are the part of the amplitude that has infinite tower of poles in s-, u- and t-channels, respectively. They are

$$A_s = 4iT_p \zeta_1{\cdot}\zeta_2\, \zeta_3{\cdot}\zeta_4 \left(\frac{\Gamma(-1-2s)\Gamma(1+s+u-t)}{\Gamma(u-t-s)} + \frac{\Gamma(-1-2s)\Gamma(1+t+s-u)}{\Gamma(t-u-s)} + \right.$$

$$\left. + \frac{\Gamma(1+s+t-u)\Gamma(1+s+u-t)}{\Gamma(2+2s)} \right) ,$$

$$
\begin{aligned}
A_u &= 4iT_p\zeta_1\!\cdot\!\zeta_3\,\zeta_2\!\cdot\!\zeta_4\left(\frac{\Gamma(1+t+u-s)\Gamma(1+s+u-t)}{\Gamma(2+2u)}+\frac{\Gamma(-1-2u)\Gamma(1+t+u-s)}{\Gamma(t-s-u)}+\right.\\
&\qquad\left.+\frac{\Gamma(-1-2u)\Gamma(1+s+u-t)}{\Gamma(s-t-u)}\right),\\
A_t &= 4iT_p\zeta_1\!\cdot\!\zeta_4\,\zeta_2\!\cdot\!\zeta_3\left(\frac{\Gamma(-1-2t)\Gamma(1+t+u-s)}{\Gamma(u-t-s)}+\frac{\Gamma(1+t+u-s)\Gamma(1+t+s-u)}{\Gamma(2+2t)}+\right.\\
&\qquad\left.+\frac{\Gamma(-1-2t)\Gamma(1+t+s-u)}{\Gamma(s-t-u)}\right),
\end{aligned}
\tag{2.3}
$$

where ζ_i's are the polarization vectors of the scalars-tachyon, and the Mandelstam variables s, t, u are the following:

$$
\begin{aligned}
s &= -\alpha'\frac{(k_1+k_2)^2}{2},\\
t &= -\alpha'\frac{(k_2+k_3)^2}{2},\\
u &= -\alpha'\frac{(k_1+k_3)^2}{2}.
\end{aligned}
\tag{2.4}
$$

Using conservation of momenta, one finds that they satisfy the relation

$$
s+t+u = -\alpha'\frac{\left(\sum_{i=1}^4 k_i^2\right)}{2},
\tag{2.5}
$$

where $k^2 = 0$ for on-shell scalar and $k^2 = 1/\alpha'$ for the on-shell tachyon. We have also normalized the amplitude (2.3) by the factor $2iT_p$ to have the complete agreement with the tachyonic DBI action including the normalization of fields. Note that if one dose not use on-shell conditions for the momenta, the amplitude (2.3) will be invariant under $\mathrm{SO}(p+1)\times\mathrm{SO}(27-(p+1))$. Also the amplitude A_s is symmetric under $1\leftrightarrow 2, 3\leftrightarrow 4$, the amplitude A_u is symmetric under $1\leftrightarrow 3, 2\leftrightarrow 4$, the amplitude A_t is symmetric under $1\leftrightarrow 4, 2\leftrightarrow 3$, and the whole amplitude A is symmetric under interchanging any particle label. Comparing this amplitude with the corresponding amplitude in the supestring theory [24], one observes the following relation

$$
\begin{aligned}
A_s^{\text{bosonic string}} &= \frac{-1}{1+2s}A_s^{\text{supestring}},\\
A_u^{\text{bosonic string}} &= \frac{-1}{1+2u}A_u^{\text{supestring}},\\
A_t^{\text{bosonic string}} &= \frac{-1}{1+2t}A_t^{\text{supestring}}.
\end{aligned}
$$

Extension of the scattering amplitude (2.3) to the noncommutative case is very easy. One should add just some phase factors to the amplitude, and use the open string metric in (2.4) [28]. The above relation between the amplitude in bosonic and super string theory remains the same. So one may use the noncommutative results in the superstring theory [24] and just add the extra tachyonic pole to find the result in the bosonic theory.

3. The α' expansion and effective action

Now we would like to expand this amplitude at the limit $\alpha' \to 0$. To do this, we should analytically continue the amplitude to the off-shell physics, e.g.,the momenta do not satisfy the on-shell condition, then expand the gamma functions. After finishing with the α' expansion we go back to the on-shell physics. In this way, one finds massless pole and infinite number of contact terms. It was shown in [24] that for the non-BPS brane of supestring theory, the massless pole and contact terms of noncommutative tachyonic DBI action are produced by the universal string theory amplitude expanded at $\alpha' \to 0$.

The amplitudes A_s, A_u can be read from the amplitude A_t by changing the states label, i.e., replacing (1432) $\to$ (1324) one gets A_u, and replacing (1423) $\to$ (1234) one gets A_s. So we continue our discussion only for the A_t amplitude. Off-shell extension of A_t is

$$A_t^{\rm off} = 4iT_p\zeta_1\cdot\zeta_4\,\zeta_2\cdot\zeta_3\Bigg(\bigg(\frac{\Gamma(-1-2t)\Gamma(1+t+u-s)}{\Gamma(u-s-t)}+\frac{\Gamma(1+t+u-s)\Gamma(1+s+t-u)}{\Gamma(2+2t)}+ \\ +\frac{\Gamma(-1-2t)\Gamma(1+t+s-u)}{\Gamma(s-u-t)}-3F_t\bigg)+3F_t\Bigg)\,, \tag{3.1}$$

where the function $F_t(k_1,k_2,k_3,k_4)$ has to cancel the non-desirable α'^2 order terms resulting from expansion of the gamma functions in the low energy limit, i.e., the first parenthesis must be independent of k_i^2 for any $i=1,2,3,4$.

Aside from the tachyonic pole in the gamma functions in (3.1), these functions have the following expansion at $\alpha' \to 0$:

$$\begin{aligned}
\frac{\Gamma(-1-2t)\Gamma(1+t+u-s)}{\Gamma(u-s-t)} &= \frac{-1}{1+2t}\left(\frac{1}{2}+\frac{s-u}{2t}-\frac{\pi^2}{6}\left((s-u)^2-t^2\right)+\cdots\right),\\
\frac{\Gamma(-1-2t)\Gamma(1+t+s-u)}{\Gamma(s-u-t)} &= \frac{-1}{1+2t}\left(\frac{1}{2}+\frac{u-s}{2t}-\frac{\pi^2}{6}\left((s-u)^2-t^2\right)+\cdots\right),\\
\frac{\Gamma(1+t+u-s)\Gamma(1+s+t-u)}{\Gamma(2+2t)} &= \frac{-1}{1+2t}\left(-1-\frac{\pi^2}{6}\left((s-u)^2-t^2\right)+\cdots\right),
\end{aligned} \tag{3.2}$$

where dots represent terms that have the Madelstam variables at lest cubic. Expansion of the tachyonic pole is also $1/(1+2t) = 1-2t+O(t^2)$. Unlike the noncommutative case that each gamma function carries a phase factor [26, 24], the gamma functions in the commutative case (3.1) have no such factors. Hence, the constant and the massless pole terms in (3.2) will be canceled when inserted into (3.1). Up to terms of second order of the Mandelstam variables, one then left with exactly the same terms that appears in the superstring case. So function F_t is exactly the one appears in the superstring theory [26],

$$F_t = -\frac{\alpha'\pi^2}{6}\Bigg(-\frac{\alpha'}{4}\left(\sum_{i=1}^{4}k_i^2\right)^2 - t\sum_{i=1}^{4}k_i^2 - \frac{\alpha'}{2}(k_1^2+k_4^2)(k_2^2+k_3^2)+ \\ +\frac{\alpha'}{2}(k_1^2+k_2^2)(k_3^2+k_4^2)+\frac{\alpha'}{2}(k_1^2+k_3^2)(k_2^2+k_4^2)\Bigg)\,. \tag{3.3}$$

We have only rearranged terms to have the symmetry $1 \leftrightarrow 4$, $2 \leftrightarrow 3$ manifest. Now at this point one should restrict the momenta to the on-shell to return to the on-shell amplitude. The resulting amplitude is

$$A_t = -12iT_p\, \zeta_1\cdot\zeta_4\, \zeta_2\cdot\zeta_3 \times \tag{3.4}$$
$$\times \left(\frac{\pi^2\alpha'^2}{3}(k_2\cdot k_3)(k_1\cdot k_4) - \frac{\pi^2\alpha'^2}{3}(k_1\cdot k_2)(k_3\cdot k_4) - \frac{\pi^2\alpha'^2}{3}(k_1\cdot k_3)(k_2\cdot k_4) - F_t^{\rm on}\right),$$

plus some other terms that are at least cubic in the Mandelstam variables. These terms are related to the higher derivative terms in the field theory. Note that for the four scalar case, the higher derivative terms are also of higher power of α' relative to the terms in (3.4). However, when one consider four tachyon amplitude, these higher derivative terms might be of the same order of α' as those appearing in (3.4), e.g.,$(\partial_a T\partial^a T)^2$ and $T^2\partial_a\partial_b T\partial^a\partial^b T$ are both of the same order of α'. However, the second one is very small relative to the first one for slowly varying tachyon field, in which we are nor interested in this paper. In the above couplings $F_t^{\rm on}$ means F in which the momenta are on-shell, i.e.,

$$\begin{aligned}
F_t^{\rm on} &= 0\,; \qquad \text{for} \quad \Phi_1\Phi_2\Phi_3\Phi_4\,,\\
F_t^{\rm on} &= -\frac{\pi^2}{3}\alpha' k_2\cdot k_3\,; \qquad \text{for} \quad T_1\Phi_2\Phi_3T_4\,,\\
F_t^{\rm on} &= -\frac{\pi^2}{3}(\alpha' k_2\cdot k_3 + \alpha' k_1\cdot k_4 + 1)\,; \qquad \text{for} \quad T_1T_2T_3T_4\,.
\end{aligned} \tag{3.5}$$

As mentioned before, apart from the on-shell function $F_t^{\rm on}$, the amplitude (3.4) is the same for scalars and the tachyons. So the field theory consistent with this part must contain both scalar and the tachyon in the same footing. However, $F_t^{\rm on}$ is different for the scalars and for the tachyons. So this term is responsible for the extra terms that the field theory produces due to the tachyon potential. Now it is easy to replace the values of $F_t^{\rm on}$ into (3.4), add the contribution from A_s, A_u, i.e., $A = A_t + A_s + A_u$, and compare the whole amplitude with the tachyonic action (1.1). For the case that the four states are tachyons, the amplitude simplifies to:

$$\begin{aligned}
A = -16\pi^2 iT_p\zeta_1^{26}\zeta_2^{26}\zeta_3^{26}\zeta_4^{26}\Big(&-\frac{\alpha'^2}{4}(k_2\cdot k_3)(k_1\cdot k_4) - \frac{\alpha'^2}{4}(k_1\cdot k_2)(k_3\cdot k_4) - \frac{\alpha'^2}{4}(k_1\cdot k_3)(k_2\cdot k_4) + \\
&+\frac{\alpha'}{4}(k_2\cdot k_3 + k_1\cdot k_4 + k_1\cdot k_3 + k_2\cdot k_4 + k_1\cdot k_2 + k_3\cdot k_4) + \frac{3}{4}\Big)\,.
\end{aligned} \tag{3.6}$$

Compare it with the following expansion of action (1.1):

$$\begin{aligned}
S = -T_p\int d^{p+1}x\Big(&1 - \pi T^2 + \beta T^4 + \\
&+\pi\alpha'(\partial_a T\partial^a T) - \pi^2\alpha' T^2(\partial_a T\partial^a T) - \frac{\pi^2\alpha'^2}{2}(\partial_a T\partial^a T)^2 + \cdots\Big)\,,
\end{aligned} \tag{3.7}$$

where the constant β is the coefficient of T^4 in the tachyon potential. One can easily observe that the term with four derivatives reproduces exactly the four momentum contact terms in (3.6), term with two derivatives reproduces the two momentum contact terms, and the T^4 term produces the constant term in (3.6) provided that $\beta = \pi^2/2$. So the even part of the tachyon potential expanded around its maximum, i.e., around $T_{max} = 0$, has the following expansion:

$$V(T) = 1 - \pi T^2 + \frac{\pi^2}{2}T^4 + O(T^6)\,. \tag{3.8}$$

This expansion is consistent with the potential (1.3) with $\alpha = 1$.

Consider the following observations: 1) the off-shell S-matrix elements involving the transverse scalars and tachyon are invariant under $\mathrm{SO}(p+1)\times\mathrm{SO}(d+1-(p+1))$ where d is critical dimension of theory, 2) the kinetic term of both scalar and tachyon appears in the same footing in the effective action (1.1). Using these, one may suspect that the tachyon, like the scalar field, represents a new physical dimension in the low energy physics. The idea that tachyon might represent a new dimension was remarked also in [29, 17].[4] Using this idea one may try to write a covariant action as:

$$S = -T_p \int d^{p+1}\sigma V(X^2)\sqrt{-\det(P[\eta_{ab}])}\,, \tag{3.9}$$

where X^A is the (d+1)-dimensional contra-variant vector, and the pull-back is

$$P[\eta_{ab}] = \eta_{AB}\frac{\partial X^A}{\partial\sigma^a}\frac{\partial X^B}{\partial\sigma^b}\,.$$

This action has global $\mathrm{SO}(d+1)$ symmetry, and the gauge symmetry of world-volume coordinate transformation. Using this latter symmetry, one can chose the static gauge, i.e., $X^a = \sigma^a$. Letting $X^i = 2\pi\alpha'\Phi^i\,, X^{d+1} = 2\pi\alpha' T$, the pull-back becomes

$$P[\eta_{ab}] = \eta_{ab} + 2\pi\alpha'\partial_a\Phi^i\partial_b\Phi_i + 2\pi\alpha'\partial_a T\partial_b T\,.$$

The action in this gauge has global $\mathrm{SO}(p+1)\times\mathrm{SO}(d+1-(p+1))$ symmetry, however, because the actual on-shell potential depends only on the $(d+1)$-th coordinate, i.e., $V(X^2) \to V(T^2)$, this symmetry is broken to $\mathrm{SO}(p+1)\times\mathrm{SO}(d-(p+1))$ in the on-shell physics.

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